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The weak-coupling collective contribution to odd–odd nuclear magnetic moments

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Abstract. It is found for the weak-coupling case of the Bohr and Mottelson collective model, that the inclusion in the Landé formula single particle g factors of core rotation effects gives an incorrect contribution to the odd–odd nuclear moment. A correction term is proposed.

1. Introduction

The use of adjacent empirical odd-A g factors in the Landé formula for the magnetic dipole moment in odd–odd nuclei (Schmidt 1937) is generally assumed to make adequate allowance for most contributions which cause deviations from the Schmidt moment (Schwartz 1953). This is certainly the case for effects which influence the neutron and proton intrinsic orbital and spin g factors such as meson quenching (Miyazawa 1951, de-Shalit 1951, Bloch 1951) and the momentum-dependent force (Miyazawa 1951). Caine (1956), however, has shown that residual interaction induced first-order configuration mixing (Blin-Stoyle 1953, Arima and Horie 1954) requires a correction to the moment to allow for admixed single-particle configurations with j_p and j_n different to those in zeroth order, since these momenta are no longer sharp in the two-particle state. This correction is of order $0{\cdot}1\ \mu_N$.

In the collective model of Bohr and Mottelson (1953) there is a change in the single-particle magnetic moment due to coupling at the nuclear surface between the particle and the core excitations (photons). It was originally noted by these authors that the inclusion of a collective phonon term acts to shift the shell model moments of odd-A nuclei inward from the Schmidt limits in accordance with experimentally observed trends. An extensive and systematic comparison of experimentally determined odd-A moments with those calculated using one- and two-phonon state admixtures has more recently been made by Kisslinger and Sorenson (1963). In the region of the closed proton shell at $Z = 28$ up to $Z = 33$ the additional terms due to the phonon coupling in each case greatly improved the shell model moment calculated using first-order configuration mixing alone. Similar results were obtained for near spherical odd-A nuclei around $Z = 50$, especially from Ag ($Z = 47$) to I ($Z = 53$). For most other nuclei however the inclusion of phonon terms still served to shift the shell model moment towards the experimental value. The Kisslinger and Sorenson odd-A moment study showed in addition that the calculated two-phonon contribution was generally less than 10% of the single-phonon term.

In all such odd-A moment calculations there is some uncertainty as to the size of the intrinsic nuclear g factors since values for the meson quenching and momentum-dependent force contributions are difficult to estimate. For *odd–odd* moments calculated using the Schwartz empirical rule such intrinsic effects are mostly accounted for as mentioned earlier. If there exists a collective model term in addition to the empirically determined odd–odd moment then its calculation and a comparison with experimental moments might provide a sensitive test for the phonon model. Such a determination of this phonon contribution is presented here.

In considering the odd–odd nucleus it is reasonable to assume that both the magnitude of the even–even core rotation (phonon angular momentum) and the strengths of the particle–core interactions are the same as in the adjacent odd-A nuclides. However, it is the purpose of this paper to point out that the relative directions of the momentum vectors in the odd–odd nucleus imply a collective contribution to μ different from that given by the use of adjacent odd-A empirical g factors in the Landé formula. A correction term is required which is typically the same size as the configuration mixing term.

2. Calculation

2.1. Weak-coupling eigenstates

In an odd-A nucleus, the collective model proposes a total spin $\boldsymbol{J} = \boldsymbol{j}+\boldsymbol{R}$ where $\boldsymbol{j}$ is the single-particle total angular momentum and $\boldsymbol{R}$ is the core rotation. $\boldsymbol{J}$ is now a good quantum number instead of $\boldsymbol{j}$. The wavefunction is expressed as (Bohr and Mottelson 1953)

$$|\psi\rangle = \alpha|j, 00; J = j\rangle + \sum_{j\lambda} \beta_{j\lambda}|j, N\lambda; J = j\rangle \tag{1}$$

where λ is the rank of the tensor describing the collective distortion and N is the number of phonons. The following discussion is restricted to first excited state admixtures and so the ground state is taken to be

$$|\psi\rangle = \alpha|j, 00; J = j\rangle + \beta|j, 12; J = j\rangle \tag{2}$$

with $\alpha^2+\beta^2 = 1$ and β given by the first-order perturbation expression

$$\beta = \frac{\langle j, 00; J = j|k(r)\Sigma_\mu a_{2\mu} Y_2^\mu(\theta, \phi)|j, 12; J = j\rangle}{\hbar\omega} \tag{3}$$

$\hbar\omega$ is the phonon energy and the other symbols have their usual meanings (Bohr and Mottelson 1953). It may be noted that β can be written $q\langle j\|Y_2\|j\rangle$ where $\langle j\|Y_2\|j\rangle$ is (for $j \geqslant \frac{1}{2}$) a number nearly independent of j (Preston 1962) and q depends on the strength of the surface coupling. The zeroth-order state, $|j, 00: J = j\rangle$, has been called the 'quasiparticle' state (Kisslinger and Sorenson 1963). The vector coupling is illustrated in figure 1(a).

The odd–odd nucleus is now treated similarly. In weak coupling it is reasonable to take the internucleon coupling strength to be greater than the individual particle–core perturbations,

$$k(r_\mathrm{p}) \sum_\mu a_{2\mu} Y_2^\mu(\theta_\mathrm{p}, \phi_\mathrm{p}) \qquad \text{and} \qquad k(r_\mathrm{n}) \sum_\mu a_{2\mu} Y_2^\mu(\theta_\mathrm{n}, \phi_\mathrm{n}).$$

The good quantum number for the two-particle state is $\boldsymbol{J} = \boldsymbol{j}_\mathrm{p}+\boldsymbol{j}_\mathrm{n}+\boldsymbol{R}$ whilst in zeroth

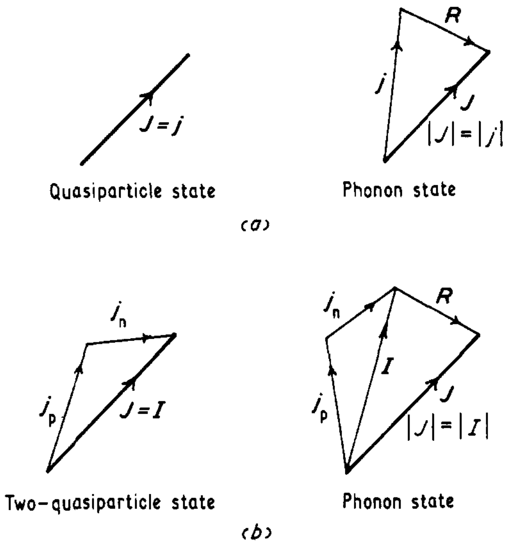

Figure 1. The coupling of the angular momenta in the admixed collective states. The odd-A scheme is shown in (a) and the odd–odd scheme in (b).

order $\boldsymbol{I} = \boldsymbol{j}_p + \boldsymbol{j}_n$. The odd–odd vector coupling is illustrated in figure 1(b) and the ground-state wavefunction is written

$$|\Psi\rangle = \alpha'|I, 00; J = I\rangle + \beta'|I, 12; J = I\rangle \tag{4}$$

with $\alpha'^2 + \beta'^2 = 1$.

If the perturbation is taken to be the sum of the particle–core interactions then

$$\beta' = (\hbar\omega)^{-1}\left\langle I, 00; J = I \left| k(r_p)\sum_\mu a_{2\mu} Y_2^\mu(\theta_p, \phi_p) + k(r_n)\sum_\mu a_{2\mu} Y_2^\mu(\theta_n, \phi_n)\right| I, 12; J = I \right\rangle. \tag{5}$$

The scalar operator $k(r)\Sigma_\mu a_{2\mu} Y_2^\mu$ operates in the phonon space and single-particle space and must be evaluated in the coupled representation. For the special case of equation (5) it is easy to show that

$$\beta' = q_p\langle j_p \| Y_2 \| j_p\rangle[(2I+1)(2j_p+1)]^{1/2}(-1)^{I+j_p+j_n}\begin{Bmatrix} j_n & j_p & I \\ 2 & I & j_p \end{Bmatrix}$$
$$+ q_n\langle j_n \| Y_2 \| j_n\rangle[(2I+1)(2j_n+1)]^{1/2}(-1)^{I+j_p+j_n}\begin{Bmatrix} j_p & j_n & I \\ 2 & I & j_n \end{Bmatrix}, \tag{6}$$

so that

$$\beta' = \beta'_p + \beta'_n \tag{7}$$

where

$$\beta'_p = (-1)^{I+j_p+j_n}[(2I+1)(2j_p+1)]^{1/2}\begin{Bmatrix} j_n & j_p & I \\ 2 & I & j_p \end{Bmatrix}\beta_p$$

and

$$\beta'_n = (-1)^{I+j_p+j_n}[(2I+1)(2j_n+1)]^{1/2} \begin{Bmatrix} j_p & j_n & I \\ 2 & I & j_n \end{Bmatrix} \beta_n. \tag{8}$$

β_p and β_n are the single-particle phonon admixture amplitudes and the symbols in curly brackets are 6-j symbols.

2.2. *The magnetic moment*

For the vector operator $\boldsymbol{j} = |j|\hat{\boldsymbol{j}}, \hat{\boldsymbol{j}}$ is a unit vector and $|j| = [j(j+1)]^{1/2}$ where $j(j+1)$ is the eigenvalue of the operator $\boldsymbol{j}^2$ and j is the angular momentum of the state (Messiah 1964). One may then define a total magnetic moment $\boldsymbol{\mu} = g\boldsymbol{j}$ where g is the state g factor. The magnitude of $\boldsymbol{\mu}$ is $\mu = g[j(j+1)]^{1/2}$ and its expectation value (eg in a magnetic field) is $\mu_z = gj = \mu(j/|j|)$. The total odd-A moment (taking, for example, the odd proton case) is (Bohr and Mottelson 1953)

$$\begin{aligned} \mu^p(\text{odd}) &= \langle\psi|\hat{\boldsymbol{J}}_p \cdot (g_p\boldsymbol{j}_p + g_R\boldsymbol{R})|\psi\rangle \\ &= \langle\psi|\hat{\boldsymbol{J}}_p \cdot (g_p\boldsymbol{J}_p - (g_p - g_R)\boldsymbol{R})|\psi\rangle \\ &= g_p|j_p| - (g_p - g_R)\langle\psi|\boldsymbol{R} \cdot \hat{\boldsymbol{J}}_p|\psi\rangle. \end{aligned} \tag{9}$$

g_p allows for all contributions to the magnetic moment apart from collective effects and g_R is the charge to mass ratio Z/A.

It can be shown similarly for odd–odd moments that

$$\mu(\text{odd–odd}) = g_I|I| - (g_I - g_R)\langle\Psi|\boldsymbol{R} \cdot \hat{\boldsymbol{J}}|\Psi\rangle. \tag{10}$$

One therefore obtains

$$\mu^p(\text{odd}) = g_p|j_p| - (g_p - g_R)\beta_p^2\frac{3}{|j_p|}. \tag{11}$$

and

$$\mu(\text{odd–odd}) = g_I|I| - (g_I - g_R)(\beta'_p + \beta'_n)^2\frac{3}{|I|}. \tag{12}$$

However the empirical Landé moment is

$$\mu(\text{emp}) = \langle\hat{\boldsymbol{I}} \cdot (g_p(\text{emp})\boldsymbol{j}_p + g_n(\text{emp})\boldsymbol{j}_n)\rangle$$

where

$$g_p(\text{emp}) = \mu_z^p(\text{odd})/j_p$$

whence

$$\mu(\text{emp}) = g_I|I| - (g_p - g_R)\beta_p^2\frac{3}{|I|}\left(1 + \frac{\boldsymbol{j}_p \cdot \boldsymbol{j}_n}{j_p(j_p+1)}\right) - (g_n - g_R)\beta_n^2\frac{3}{|I|}\left(1 + \frac{\boldsymbol{j}_p \cdot \boldsymbol{j}_n}{j_n(j_n+1)}\right) \tag{13}$$

One may now write for the measured moment (expectation value)

$$\mu_z(\text{odd–odd}) = \mu_z(\text{emp}) + \Delta\mu_z(\text{R})$$

or, allowing for the configuration mixing terms $\Delta\mu_z(\text{p}), \Delta\mu_z(\text{n})$

$$\mu_z(\text{odd–odd}) = \mu_z(\text{emp}) + \Delta\mu_z(\text{p}) + \Delta\mu_z(\text{n}) + \Delta\mu_z(\text{R}) \tag{14}$$

where

$$\begin{aligned}\Delta\mu_z(\mathrm{R}) = \frac{3}{I+1}\Bigg\{&\beta_{\mathrm{p}}^2\Bigg[(g_{\mathrm{p}}-g_{\mathrm{R}})\left(1+\frac{\boldsymbol{j}_{\mathrm{p}}\cdot\boldsymbol{j}_{\mathrm{n}}}{j_{\mathrm{p}}(j_{\mathrm{p}}+1)}\right)-(g_{\mathrm{I}}-g_{\mathrm{R}})(2I+1)(2j_{\mathrm{p}}+1)\begin{Bmatrix} j_{\mathrm{n}} & j_{\mathrm{p}} & I \\ 2 & I & j_{\mathrm{p}}\end{Bmatrix}^2\Bigg]\\ &+\beta_{\mathrm{n}}^2\Bigg[(g_{\mathrm{n}}-g_{\mathrm{R}})\left(1+\frac{\boldsymbol{j}_{\mathrm{p}}\cdot\boldsymbol{j}_{\mathrm{n}}}{j_{\mathrm{n}}(j_{\mathrm{n}}+1)}\right)-(g_{\mathrm{I}}-g_{\mathrm{R}})(2I+1)(2j_{\mathrm{n}}+1)\begin{Bmatrix} j_{\mathrm{p}} & j_{\mathrm{n}} & I \\ 2 & I & j_{\mathrm{n}}\end{Bmatrix}^2\Bigg]\\ &-2\beta_{\mathrm{p}}\beta_{\mathrm{n}}(g_{\mathrm{I}}-g_{\mathrm{R}})(2I+1)[(2j_{\mathrm{p}}+1)(2j_{\mathrm{n}}+1)]^{1/2}\begin{Bmatrix} j_{\mathrm{n}} & j_{\mathrm{p}} & I \\ 2 & I & j_{\mathrm{p}}\end{Bmatrix}\begin{Bmatrix} j_{\mathrm{p}} & j_{\mathrm{n}} & I \\ 2 & I & j_{\mathrm{n}}\end{Bmatrix}\Bigg\}\end{aligned} \tag{15}$$

The term $\Delta\mu_z(\mathrm{R})$ arises because the coupling of the rotational motion to the resultant individual particle motion in the odd–odd nucleus differs from that in adjacent odd-A nuclei. Thus the empirical g factor derived from these adjacent nuclei requires correction.

3. Conclusion

The proposed correction term, $\Delta\mu_z(\mathrm{R})$, may be estimated using state admixture amplitudes $\beta_{\mathrm{p}}, \beta_{\mathrm{n}}$ given by the Kisslinger and Sorenson 'pairing plus quadrupole' model or the original Bohr and Mottelson collective model. $\Delta\mu_z(\mathrm{R})$ is typically 0·1 μ_{N} and its sign is sensitive to the proton–neutron coupling through $\boldsymbol{j}_{\mathrm{p}}\cdot\boldsymbol{j}_{\mathrm{n}}$, g_{I} and the 6-j symbols.

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